Practice USAPhO M

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all problems. Each problem is worth an equal number of points, with a total point value of 75. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete all problems. Each problem is worth an equal number of points, with a total point value of 75. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your AAPT ID number, your proctor's AAPT ID number, the question number, and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

  Student AAPT ID #
  Proctor AAPT ID #
  A1 – 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest.

Possibly Useful Information. You may use this sheet for both parts of the exam.

$g = 9.8$ N/kg	$G = 6.67 \times 10^{-11}$ N $\cdot$ m^2/kg^2
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9$ N $\cdot$ m^2/C^2	$k_\text{m} = \mu_0/4\pi = 10^{-7}$ T $\cdot$ m/A
$c = 3.00 \times 10^8$ m/s	$k_\text{B} = 1.38 \times 10^{-23}$ J/K
$N_\text{A} = 6.02 \times 10^{23}$ (mol)$^{-1}$	$R = N_\text{A} k_\text{B} = 8.31$ J/(mol $\cdot$ K)
$\sigma = 5.67 \times 10^{-8}$ J/(s $\cdot$ m^2 $\cdot$ K^4)	$e = 1.602 \times 10^{-19}$ C
1 eV $= 1.602 \times 10^{-19}$ J	$h = 6.63 \times 10^{-34}$ J $\cdot$ s $= 4.14 \times 10^{-15}$ eV $\cdot$ s
$m_e = 9.109 \times 10^{-31}$ kg $= 0.511$ MeV/c^2	$(1+x)^n \approx 1 + nx$ for $\lvert x\rvert \ll 1$
$\sin\theta \approx \theta - \frac{1}{6}\theta^3$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ for $\lvert\theta\rvert \ll 1$

Part A

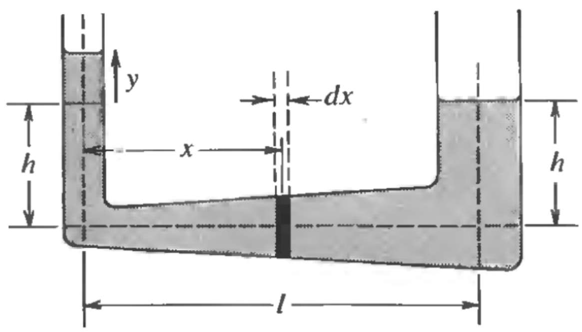
Question A1

Two masses, m_1 and m_2, attached to equal length massless strings, are hanging side-by-side just in contact with each other. Mass m_1 is swung out to the side to a point having a vertical displacement 20 cm above mass m_2. It is released from rest and collides elastically with the stationary hanging mass m_2. Each of the masses is observed to rise to the same height following the collision. Neglect the volumes of the masses.

1. Find the numerical value of this height.
2. The masses swing back down and undergo a second elastic collision. After this collision, how high do the masses rise?
3. Suppose that the collisions are instead slightly inelastic. After a long time, how high does each mass rise in its motion, and qualitatively what is their relative position?

Question A2

A U-tube has vertical arms of radii r and $2r$, connected by a horizontal tube of length l whose radius increases linearly from r to $2r$. The U-tube contains liquid up to height h in each arm. The liquid is set oscillating, and at a given instant the liquid in the narrower arm is at a distance y above the equilibrium level.



1. Show that, to second order in y, the change in potential energy of the liquid is
$$U = \frac{5}{8} g \rho \pi r^2 y^2.$$
2. Show that, to second order in y, the kinetic energy of the liquid is
$$K = \frac{1}{4} \rho \pi r^2 \left(\ell + \frac{5}{2} h \right) \left(\frac{dy}{dt} \right)^2.$$
You may find it useful to integrate over slices dx, as shown in the figure. Ignore any nastiness at the corners, and assume $\ell \gg r$.
3. Assuming $\ell = 5h/2$, compute the period of oscillations.
4. Explain why the assumption $\ell \gg r$ is necessary to get an accurate result.

Question A3

Two stars of masses M_1 and M_2 are initially orbiting each other in a circular orbit, with relative velocity v and separation r. The first star begins slowly transferring matter to the second.

1. Show that during this process, the quantities $M_1 M_2 v^a r$ and $v^b r$ are conserved, for some values of a and b, and find these values.

2. If the mass transfer rate is μ, what is dr/dt, in terms of r, M_1, M_2, and μ?

Part B

Question B1

A man wishes to topple a very tall and thin obelisk, of height L. To do this, he wraps the end of a rope of length L around the obelisk at height h, then stands on the ground and pulls the other end as hard as he can. Assume that the rope does not slip on the obelisk, but the man can slip on the ground, with coefficient of static friction μ.

1. Explain why the man is unlikely to succeed if he attaches the rope at $h = 0$ or $h = L$.
2. To topple the obelisk, the man should maximize the torque they can exert about the obelisk's base without slipping. What is the optimal value of h?

Question B2

The bottom of the Marianas trench in the Pacific ocean is 10.9 km below sea level.

1. Estimate the pressure at the bottom of the trench, assuming the water is incompressible. The density of water at atmospheric pressure is $1025\,\mathrm{kg/m^3}$.
2. In reality, water is not incompressible. Its compressibility is described by its bulk modulus,
$$B = \rho \frac{dP}{d\rho} = 2.1 \times 10^9\,\mathrm{Pa}.$$
The bulk modulus has the same dimensions as the Young's modulus, defined as stress over strain. They're fundamentally very similar, but the bulk modulus quantifies the response to uniform pressure, while the Young's modulus quantifies the response to stress in one direction.
Find the pressure at the bottom of the trench, accounting for the compressibility of water, to within 10% accuracy.
3. A bathyscaph is a spherical diving vessel designed to descend to great depths in the ocean. Estimate the thickness of steel wall needed to withstand the pressure at the bottom of the Mariana trench, to within 10% accuracy. Assume the initial radius is 1 m, the Young's modulus of steel is $2 \times 10^{11}\,\mathrm{Pa}$, and that steel breaks down at a strain (fractional length change) of above 0.5%. (Hint: consider forces between two halves of the bathyscaph.)

Question B3

A uniform ring of mass m and radius R has a point mass of mass M attached to it. The ring is placed on the ground, with the point mass initially at its highest point, and is given an infinitesimal sideways impulse in the plane of the ring. Assume the ring does not slip.

1. Assuming the ring never loses contact with the ground, find the angular velocity ω of the ring as a function of the angle θ through which it has rotated.
2. Continuing to assume the ring never loses contact with the ground, find the vertical component of the force on the ring-mass system as a function of θ.
3. It turns out that for some range of values of m/M, the ring leaves contact with the ground at some point. What are these values? (For partial credit, you can instead prove that there *exists* a value of m/M so that the ring jumps.)