

Practice USAPhO C

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all problems. Each problem is worth an equal number of points, with a total point value of 90. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete all problems. Each problem is worth an equal number of points, with a total point value of 90. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your AAPT ID number, your proctor's AAPT ID number, the question number, and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

Student AAPT ID #
 Proctor AAPT ID #
 A1 – 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **For this exam alone, you may use a ruler and a movable hard surface, such as a portable whiteboard, an iPad, a cutting board, or anything similar. You may also use a small mass, such as a paperweight or a book.**
- In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest.

Possibly Useful Information. You may use this sheet for both parts of the exam.

$$g = 9.8 \text{ N/kg}$$

$$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

$$c = 3.00 \times 10^8 \text{ m/s}$$

$$N_A = 6.02 \times 10^{23} \text{ (mol)}^{-1}$$

$$\sigma = 5.67 \times 10^{-8} \text{ J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4)$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$m_e = 9.109 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV}/c^2$$

$$\sin \theta \approx \theta - \frac{1}{6}\theta^3 \text{ for } |\theta| \ll 1$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

$$k_m = \mu_0/4\pi = 10^{-7} \text{ T} \cdot \text{m/A}$$

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$R = N_A k_B = 8.31 \text{ J}/(\text{mol} \cdot \text{K})$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$(1 + x)^n \approx 1 + nx \text{ for } |x| \ll 1$$

$$\cos \theta \approx 1 - \frac{1}{2}\theta^2 \text{ for } |\theta| \ll 1$$

Part A

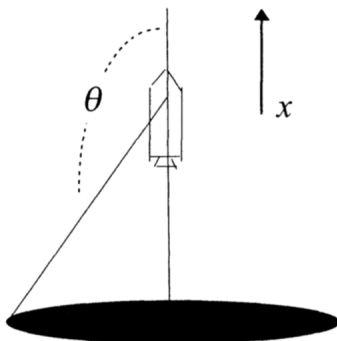
Question A1

A solid homogeneous metal ball of radius R is dropped with its lowest point a height h above the floor. It has zero initial velocity, and an initial angular velocity of $\omega \hat{\mathbf{x}}$ about its center of mass. After impact, the ball rebounds so that the maximum subsequent height of its lowest point is αh . Assume the impact time is negligible, and let the mass of the ball be m , the acceleration due to gravity be g , and the coefficient of kinetic friction between the ball and floor be μ_k .

1. Find the minimum value of ω so that the ball slips throughout the entire impact.
2. Assuming ω is above this minimum value, find the distance between the ball's first and second impact points.

Question A2

A spaceship starting at the center of the Milky Way begins to move away from it, perpendicular to its plane, at a constant speed v . If one looks out from the window of the spaceship, one sees a strangely distorted image of the galaxy. To understand this, suppose a light ray from the galaxy intercepts the spaceship at an angle θ , as shown.



In the frame of the spaceship, the angle is instead θ' .

1. Using the relativistic velocity transformations

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2}, \quad u'_y = \frac{u_y}{\gamma(1 - vu_x/c^2)}$$

find an expression for θ' .

2. Suppose light from the edge of the galaxy comes in at an angle $\theta = 150^\circ$. How fast does the spaceship have to be moving for this to correspond to an angle $\theta' = 30^\circ$? For all future parts, assume the spaceship is moving at this speed.
3. For what values of θ' can light from the galaxy be seen?
4. Suppose there are pulsars distributed throughout the galaxy, which pulse with frequency 1 Hz in their rest frames. What pulse frequencies does an observer inside the spaceship see with their eyes, from pulsars that are at the center of the galaxy, and at the edge?
5. Qualitatively, how does the image of the galaxy change as the spacecraft continues at constant speed?

Question A3

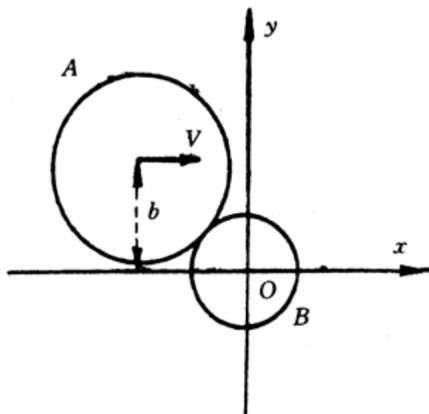
Devise and perform an experiment to find the coefficients of static friction between (1) paper and paper, and (2) paper and graphite mixture, the residue left behind when one draws on paper with a pencil, as accurately as possible. You may use a ruler, a movable hard surface, a mass, and the pencil and paper you brought to this exam.

To receive credit, you should describe the method you used and justify why you have chosen it. You do not have to do any graphing for this problem. State your results for the two coefficients of friction along with uncertainties. (Most of the credit for this question is for producing a sensible experimental method, not your final numeric results.)

Part B

Question B1

A homogeneous disc A of mass m and radius R_A moves on a plane in the \hat{x} direction with speed v . The center of the disc is at a distance b from the x -axis. It collides with a stationary homogeneous disc B whose center is initially located at the origin. Disc B has mass m and radius R_B . Both discs are initially nonrotating. The plane is frictionless, but the discs have friction with each other.



The parameter b is such that the two discs collide. Assume that:

- The velocities of the discs at their point of contact, in the direction perpendicular to the line joining their centers, are equal after the collision.
- The magnitudes of the relative velocities of the discs along the line joining their centers are the same before and after the collision.

Determine the velocities \mathbf{v}'_A and \mathbf{v}'_B of the discs after the collision.

Question B2

Consider two very long, tightly wound cylindrical solenoids with n turns per unit length, total length L , and cross-sectional areas $A_1 > A_2$. The second solenoid is placed concentrically inside the first, so that the centers of the two coils coincide. Both solenoids are hooked up to constant current sources I , so that they generate magnetic fields in the same direction.

1. Compute the total magnetic field energy. You may ignore fringe fields.

Now suppose that the second solenoid is moved a distance x to the right along its symmetry axis, so that the centers of the two solenoids no longer coincide. You may assume that $\sqrt{A_i} \ll x \ll L$.

2. Compute the total magnetic field energy when both coils are held in place at this position.
3. Find the electromotive forces \mathcal{E}_1 and \mathcal{E}_2 generated on the coils when the second one is pulled further out with a velocity v .
4. Now suppose again that both coils are held in place. Find the force F needed to hold the second coil in place, and indicate its direction.