Quantum and Classical Effects of Axion Dark Matter

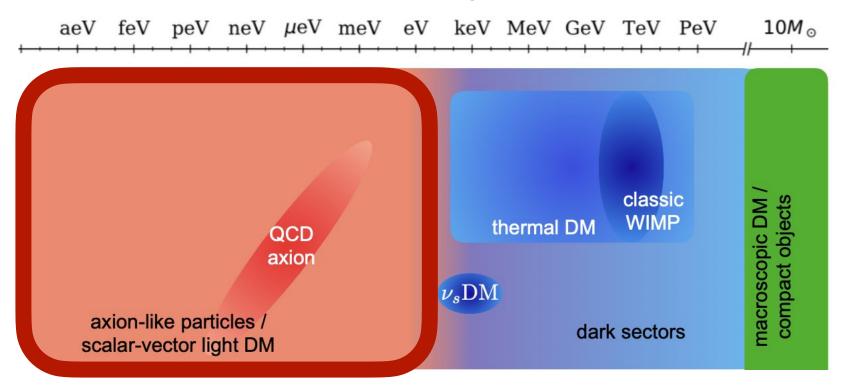
Kevin Zhou



Berkeley 4D Seminar — October 27, 2025

arXiv:2510.05198, with Yunjia Bao, Dhong Yeon Cheong, Nicolas Rodd, Joey Takach, Lian-Tao Wang

Why Search for Ultralight Dark Matter?



- Simply produced: gives right amount of dark matter with minimal cosmology
- Generic: required ultralight fields automatically appear in many models
- Minimal: requires introduction of only a single new field at low energies
- Low-hanging fruit: new experiments are needed, inexpensive, and very effective
- Bounded: only a few interactions are natural and leading in effective field theory

$$(\partial_{\mu}a)\,K^{\mu}_{
m EM}$$
 axion-photon

$$(\partial_{\mu}a)\,\bar{N}\gamma^{\mu}\gamma^5N$$

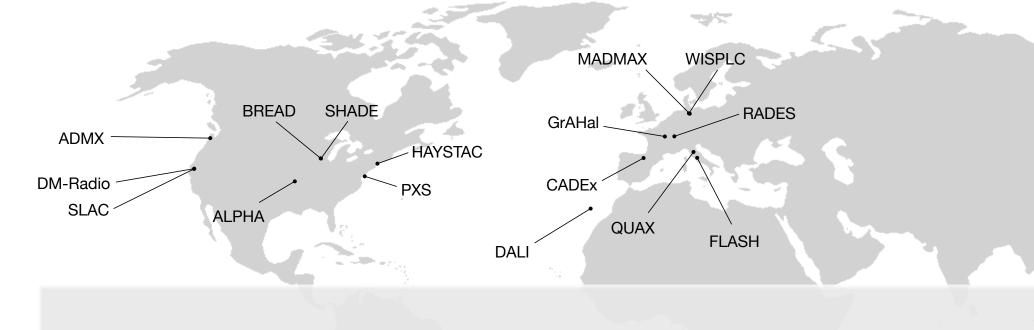
$$(\partial_{\mu}a)\,\bar{e}\gamma^{\mu}\gamma^{5}e$$

$$a\,ar{N}\sigma_{\mu\nu}\gamma^5N\,F^{\mu\nu}$$

axion-nucleon

axion-electron

axion-EDM





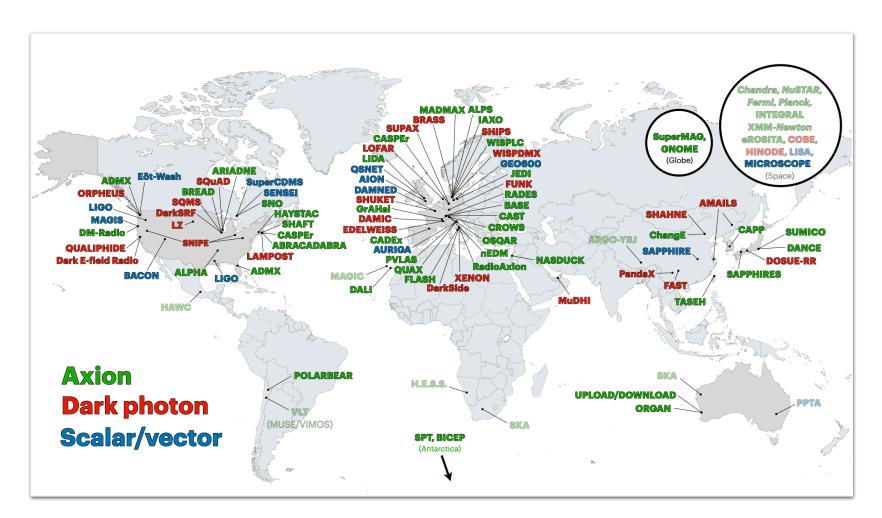
ORGAN

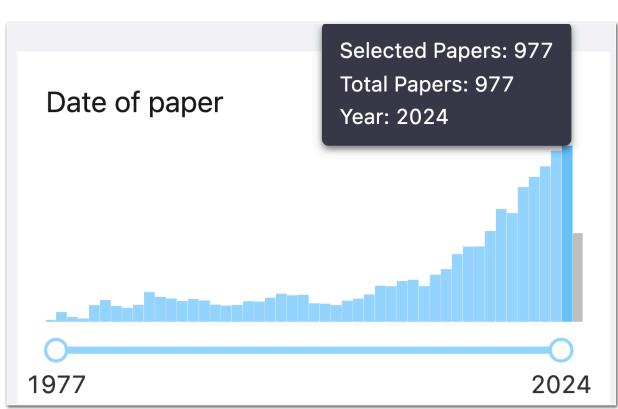
Most existing/proposed experiments target photon coupling

"Effective current" $\mathbf{J}_{\text{eff}} = g_{a\gamma\gamma}\dot{a}\mathbf{B}$ in background \mathbf{B} sources secondary, detectable electromagnetic fields

Axion Searches: Theory and Practice

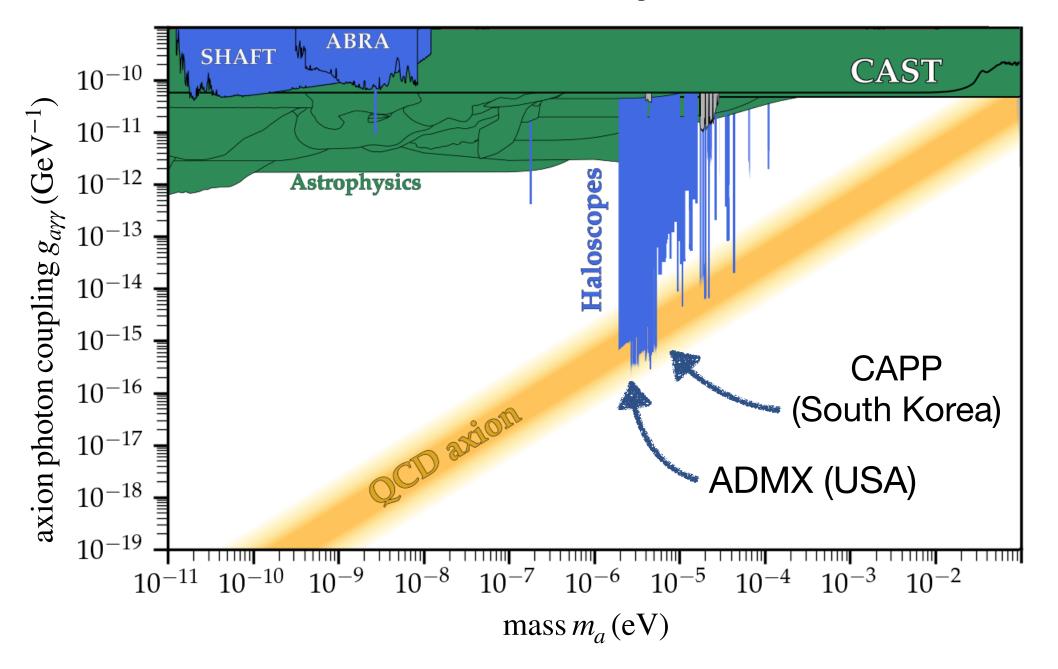
Many experimental collaborations have been formed, and (too) many papers written





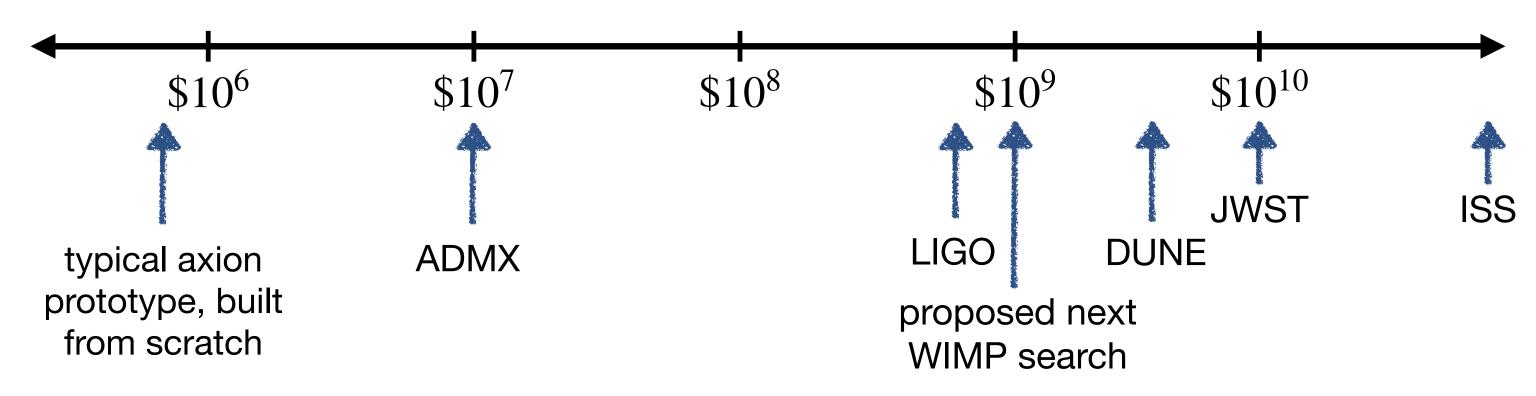
But many experiments are still only prototypes, and core experimental ideas have remained the same

Axion Searches: Theory and Practice



Currently, only the axion-photon coupling has been probed significantly, and largely by just two experiments using the same method

Axion Searches: Theory and Practice

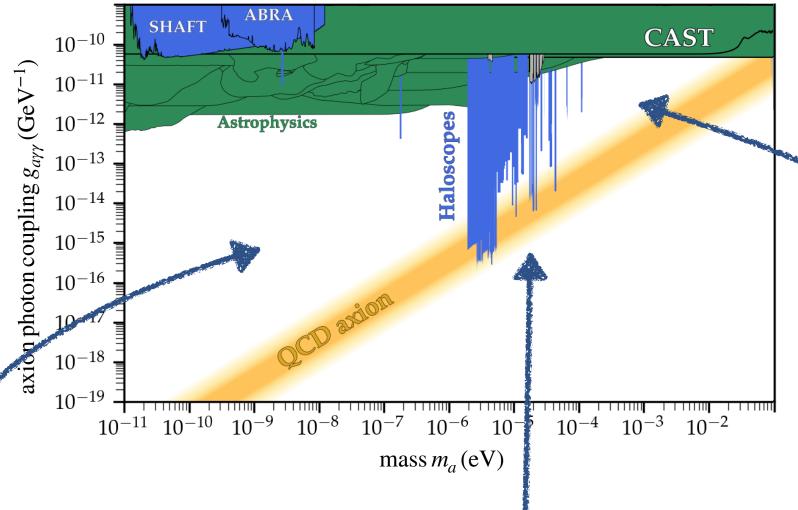


Axion experiments are "small scale" science

Strong sensitivity is within the reach of a university lab

A decisive next-generation axion experiment would be much smaller than "flagship" particle physics projects

Axion Mass Benchmarks



"high" mass
motivation: postinflation production
of QCD axion

"low" mass

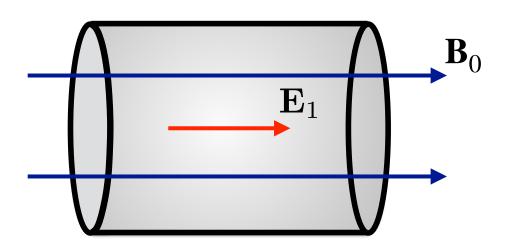
motivation: axions from grand unification/string theory or standard misalignment production of non-QCD axion

"central" mass, $f \sim \text{GHz}$

motivation: standard misalignment production of QCD axion

"Central" Mass: The Cavity Haloscope

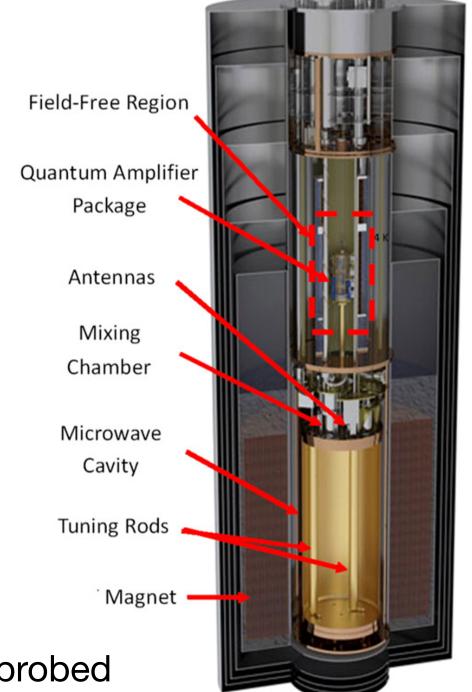
In background ${\bf B}_0$, axion drives cavity mode with profile ${\bf E}_1$



$$P_{\text{sig}} \sim (g_{a\gamma\gamma}^2 \rho_{\text{DM}}) (B_0^2 V) (Q_1/\omega_1)$$

magnetic energy in cavity

decay time of cavity mode



~10 active collaborations, ~1/3 of relevant parameter space probed

Modifying the Cavity Haloscope at "High" Mass

$$P_{\text{sig}} \sim (g_{a\gamma\gamma}^2 \rho_{\text{DM}}) (B_0^2 V) (Q_1/\omega_1)$$
 $V \propto 1/\omega_1^3$

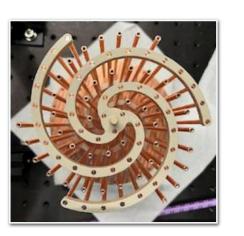
$$V \propto 1/\omega_1^3$$

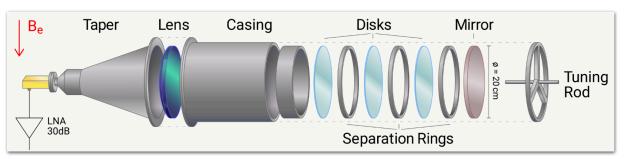
$$S_{\text{SQL}} \sim \hbar \omega_1$$

As axion mass increases, signal power decreases but SQL noise increases

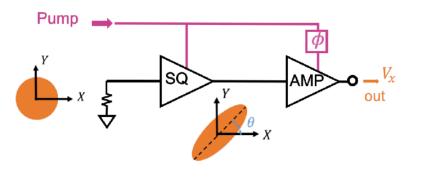
exotic design to maintain large volume

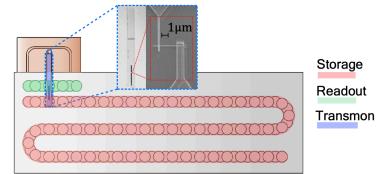






quantum measurement to evade SQL





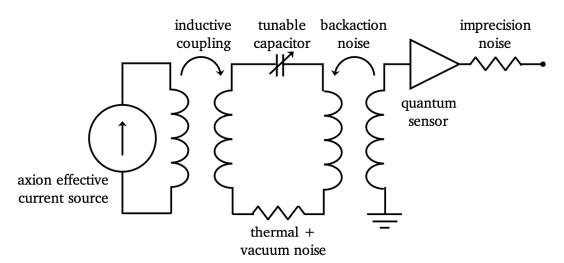
squeezed vacuum

meV single photon detection (Josephson junctions, qubits, Rydberg atoms)

Naturally suited for many small-scale efforts, and "quantum" science initiatives Potential of these approaches recognized 10+ years ago, technology rapidly maturing

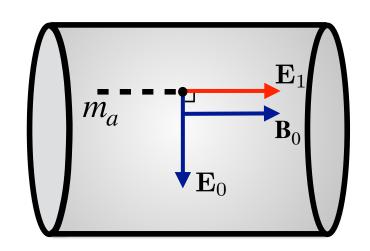
Modifying the Cavity Haloscope at "Low" Mass

A realistic cavity mode can't have frequency $m_a \ll \text{GHz}$



use LC circuit, with resonant frequency m_a great potential precision with quantum techniques magnetoquasistatic signal penalty $P_{\rm sig} \propto (m_a L)^2$ Kahn, Safdi, Thaler, PRL (2016)

LC circuit approach

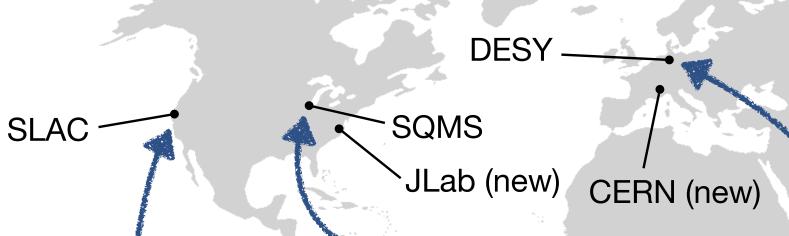


heterodyne approach

drive a cavity mode at $\omega_0 \sim \text{GHz}$ axion excites another mode at $\omega_1 = \omega_0 \pm m_a$ aided by well-developed SRF accelerator cavities noisier, but higher signal power

Berlin, D'Agnolo, ..., KZ, JHEP (2020) Berlin, D'Agnolo, Ellis, KZ, PRD (2021)

Global Status of the Heterodyne Approach



Li, KZ, Oriunno, et al. (2507.07173)









Peking

Why can the axion be treated as a classical field?



Most states, even with large N, are intrinsically quantum!

Why can the axion be treated as a classical field?

"Because the average mode occupancy
$$N \sim \frac{\rho_{\rm DM}}{m_a (m_a v_{\rm DM})^3} \sim \left(\frac{10\,{\rm eV}}{m_a}\right)^4$$
 is high"

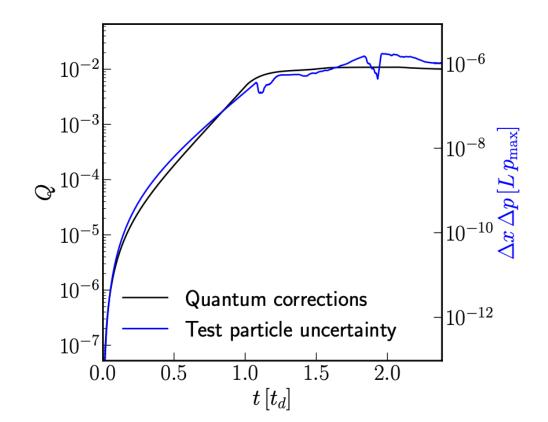


Most states, even with large N, are intrinsically quantum!

"Because misalignment produces the axion in a coherent state"



Not true (coherent oscillation ≠ coherent state), and doesn't persist



Inflation squeezes the axion state

Galactic dynamics makes deviations from classicality grow exponentially

Eberhardt et al., PRD (2024)

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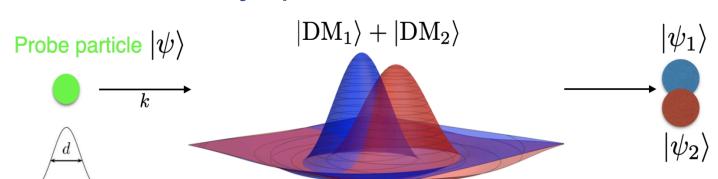
Most states, even with large N, are intrinsically quantum!

"Because misalignment produces the axion in a coherent state"



Not true (coherent oscillation ≠ coherent state), and doesn't persist

"Because intrinsically quantum states would decohere"



Allali and Hertzberg, PRL (2021)

Superposition of $|\alpha\rangle$ and $|-\alpha\rangle$ has negligible gravitational decoherence (same Newtonian potential)

Why think about this question?

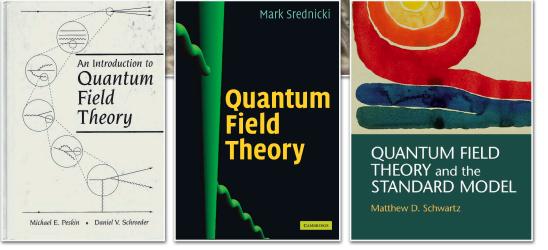




$$\frac{|\operatorname{a}\operatorname{big}\operatorname{deal}\rangle + |\operatorname{no}\operatorname{big}\operatorname{deal}\rangle}{\sqrt{2}}$$



Since 1960s, tradition in particle physics has been to avoid intermediate states



Standard particle physics texts avoid even mentioning the state of a quantum field!

The State of a Quantum Field

A quantum field is just an infinite collection of harmonic oscillators

Attempt #1: generalize number basis
$$|\psi\rangle = \sum_{n} c_n |n\rangle$$
 to $|\Psi\rangle = \sum_{n_1, n_2, \dots} c_{n_1 n_2, \dots} |n_1 n_2, \dots\rangle$

Good for collider, but far from axion DM states, classical limit unclear

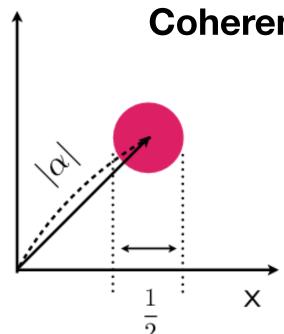
Attempt #2: generalize position basis
$$|\psi\rangle = \int dx \, \psi(x) \, |x\rangle$$
 to $|\Psi\rangle = \int \mathcal{D}\psi(\mathbf{x}) \, \Psi[\psi(\mathbf{x})] \, |\psi(\mathbf{x})\rangle$

Hard to compute with this "Schrodinger wavefunctional", mixed states even worse

Attempt #3: generalize coherent states $|\alpha\rangle$ where $a|\alpha\rangle = \alpha |\alpha\rangle$



Glauber's 2005 Nobel prize: this is the right approach!



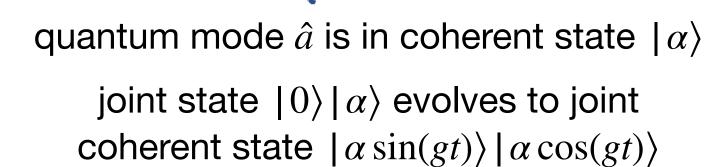
Coherent States Act Classically

coherent states have relatively welldefined values of both quadratures

Axion and cavity EM modes like coupled harmonic oscillators $H_{\text{int}} = ig (b^{\dagger}a - b a^{\dagger})$



a is a classical number α cavity state $|0\rangle$ evolves to coherent state $|\alpha \sin(gt)\rangle$



From cavity's perspective, coherent states act like classical field values!

Generalization to Arbitrary States

If coherent state corresponds to a classical field with definite value, a classical field with unknown value corresponds to the mixed state

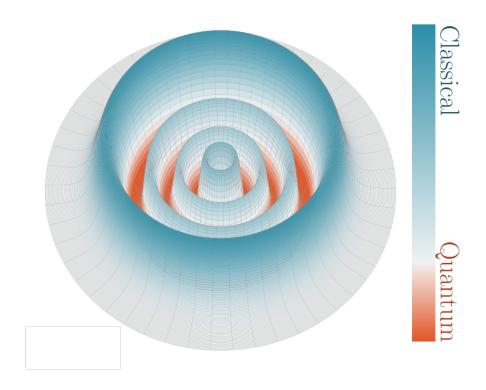
$$\rho = \int d^2 \alpha P(\alpha) |\alpha\rangle\langle\alpha|, \qquad P(\alpha) \ge 0$$

Glauber's key insight: all states can be written uniquely in this form!

However, generic quantum states require $P(\alpha) < 0$, so **not** like classical ensemble

"cat" state $|\alpha\rangle + |\alpha'\rangle$ (also with squeezed state $|\gamma\rangle$ Gaussian noise) number state $|n\rangle$

These "intrinsically quantum" states produce unique modifications of measurement statistics



Quantum Description of the Cavity Haloscope

All the axion modes are coupled to a cavity mode

$$H_{\rm int} = g_{a\gamma\gamma} B_0 \int_V d^3 \mathbf{x} \, \phi(\mathbf{x}) E_z(\mathbf{x}) \qquad E_z(\mathbf{x}) \supset ib\tilde{E}_0(\mathbf{x}) + \text{h.c.} \qquad \phi(\mathbf{x}) = \sum_{\mathbf{p}} \frac{1}{\sqrt{2\omega_p \mathcal{V}}} (a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} + \text{h.c.})$$

On resonance in interaction picture ($\omega_0 \simeq m_a$), reduces to a rotation between cavity mode and one "effective" axion mode

$$H_{\rm int}(t) \simeq ig \, (b^{\dagger} a_{\rm eff}(t) - b \, a_{\rm eff}^{\dagger}(t))$$
 $a_{\rm eff}(t) = \sum_{\mathbf{p}} C_{\mathbf{p}} \, e^{-iK_p t} \, a_{\mathbf{p}}$ $K_p \simeq \frac{p^2}{2m_a}$ $g \sim g_{a\gamma\gamma} B_0$

To compute all observables, start with effective mode's P-function, and track evolution of cavity's P-function

For simplicity: projectively measure cavity after time t_m , where $K_p t_m \lesssim 1$

Quantum State of the Effective Axion Mode

Quantum state of the axion field is joint *P*-function $\rho = \int \left(\prod_i d^2\alpha_i \, | \, \alpha_i \rangle \langle \alpha_i | \right) P(\alpha_1, \alpha_2, \ldots)$

We observe only the effective mode, whose *P*-function is

$$P_{\text{eff}}(\alpha) = \int \left(\prod_{i=1}^{\Omega} d^2 \alpha_i \right) P(\alpha_1, \alpha_2, \dots) \, \delta\left(\alpha - \sum_i C_{\mathbf{p}} e^{iK_{\mathbf{p}}t} \, \alpha_i \right)$$

If the plane wave modes are unentangled, $P(\alpha_1, \alpha_2, ...) = \prod_i P_i(\alpha_i)$

Then central limit theorem logic implies Gaussian state $P_{\rm eff}(\alpha) \propto e^{-|\alpha|^2/\langle n \rangle}$

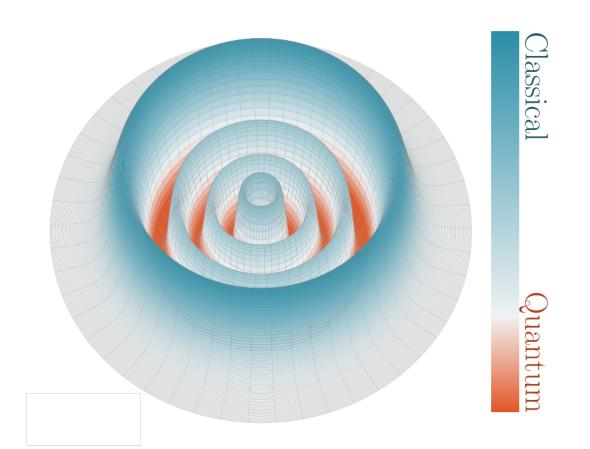
Generically makes axion behave like a classical Gaussian random field, but alternatives possible (mode entanglement, Bose-Einstein condensation)

Evolution of the Cavity State

The Hamiltonian just mixes two modes, and time evolution can be computed exactly

$$H_{\rm int} \simeq ig \, (b^{\dagger} a_{\rm eff} - b \, a_{\rm eff}^{\dagger})$$
 $P_{\rm cav}^f(\alpha) \simeq \int d^2 \beta \, \frac{P_{\rm eff}(\beta/\sqrt{\eta})}{\eta} P_{\rm cav}^i(\alpha - \beta)$ $\eta = \sin^2(gt) \ll 1$

(scaled convolution of DM state and initial cavity state)



Measurement statistics are integrals of P_{cav}^f

Any P can only be negative in O(1) regions

Then $P_{\rm cav}^f$ only negative in $O(\sqrt{\eta})$ regions

Intrinsically quantum effects highly suppressed!

Suppression of Quantum Effects

The Hamiltonian just mixes two modes, and time evolution can be computed exactly

$$H_{\rm int} \simeq ig \, (b^{\dagger} a_{\rm eff} - b \, a_{\rm eff}^{\dagger})$$
 $P_{\rm cav}^f(\alpha) \simeq \int d^2 \beta \, \frac{P_{\rm eff}(\beta/\sqrt{\eta})}{\eta} \, P_{\rm cav}^i(\alpha - \beta)$ $\eta = \sin^2(gt) \ll 1$

Conversion efficiency at critical coupling: $\eta \sim 10^{-14} \left(\frac{g_{a\gamma\gamma}}{10^{-12} \, \mathrm{GeV^{-1}}} \frac{B_0}{10 \, \mathrm{T}} \frac{Q_c}{10^6} \frac{10^{-5} \, \mathrm{eV}}{m_a} \right)^2$

Deviations from classical statistics suppressed by powers of η

Thermal noise by itself will wash out negativity unless $\eta \gtrsim \exp(-m_a/T)$, i.e.

$$T \lesssim \frac{m_a}{\log(1/\eta)} \simeq 2 \,\mathrm{mK} \left(\frac{m_a}{10^{-5} \,\mathrm{eV}}\right)$$
 (stringent but possible)

Example: Nonclassical Number Statistics

For perfect initial vacuum state, final number distribution is $p_n = \int d^2\alpha \, P_{\rm eff}(\alpha) \, |\langle n \, | \sqrt{\eta} \, \alpha \rangle \, |^2$

Example: only nonclassical states have Mandel $Q = (var(n)/\langle n \rangle) - 1 < 0$

Consider edge of sensitivity, $p_2 \ll p_1 \ll p_0 \approx 1$, so that $p_1 \approx n_c = \eta N_{\rm eff}$

axion state	Mandel $Q_{ m DM}$	value of p_2	Mandel $Q_{\rm cav}$
coherent	O	$n_c^2/2$	0
Gaussian	$N_{ m eff}$	n_c^2	n_c
number	- 1	$(n_c^2/2)(1 - \eta/n_c)$	$-\eta$

Actual Mandel Q imprinted in cavity is small, corresponds to very slightly lowered p_2

Example: Nonclassical Number Statistics

axion state	value of p_2	Mandel $Q_{\rm cav}$
coherent	$n_c^2/2$	0
Gaussian	n_c^2	n_c
number	$(n_c^2/2)(1 - \eta/n_c)$	$-\eta$

Time to discover the axion: t_m/n_c (by definition, reasonable at edge of sensitivity)

Time to distinguish coherent and Gaussian: t_m/n_c^2 (reasonable for post-discovery)

Time to observe negative
$$Q_{\rm cav}$$
: $t_m/\eta^2 \sim 10^{10}\,{\rm years}\left(\frac{10^{-12}\,{\rm GeV^{-1}}}{g_{a\gamma\gamma}}\frac{30\,{\rm T}}{B_0}\right)^4\left(\frac{10^6}{Q_c}\frac{m_a}{10^{-6}\,{\rm eV}}\right)^3$

Entanglement and Other Signatures

Isn't this just smoke and mirrors? Surely DM superposition can entangle with the cavity?

$$(|\alpha\rangle + |-\alpha\rangle)|0\rangle_{\text{cav}} \qquad |\alpha\cos gt\rangle |\alpha\sin gt\rangle_{\text{cav}} + |-\alpha\cos gt\rangle |-\alpha\sin gt\rangle_{\text{cav}}$$

Definitely exotic, but we can only measure the cavity state! Tracing out DM gives

$$\rho_{\text{cav}} \sim \frac{1}{2} \begin{pmatrix} 1 & \langle -\alpha \cos gt | \alpha \cos gt \rangle \\ \langle \alpha \cos gt | -\alpha \cos gt \rangle & 1 \end{pmatrix}$$
 almost identical to 50/50 mixture

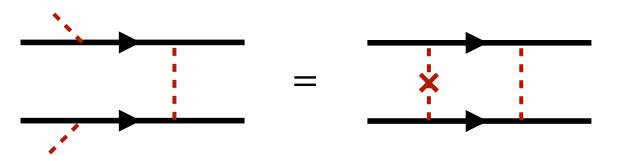
More generally, our framework can handle exotic quantum measurement protocols, many nonclassicality measures, multiple cavities, other DM interactions...

Nonclassical effects always extremely suppressed!

Treatment of axion as classical field could be wrong, but still **effectively** correct

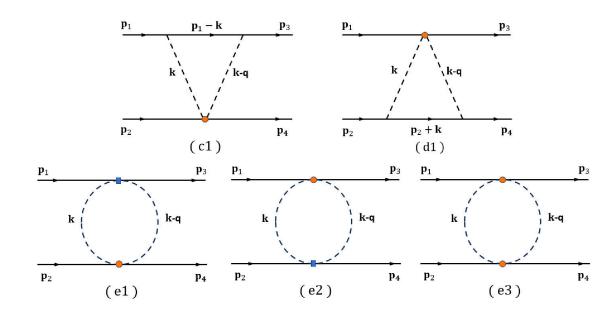
"DM Background-Induced" Forces are Classical

Tree-level coupling to external axions is formally a background-corrected loop diagram



Recent claim: axion-mediated potential can become 1/r, spin independent

Looks quantum, but isn't: short classical calculation gives exact same result!



$$(\partial^2 + m_a^2) a = J$$

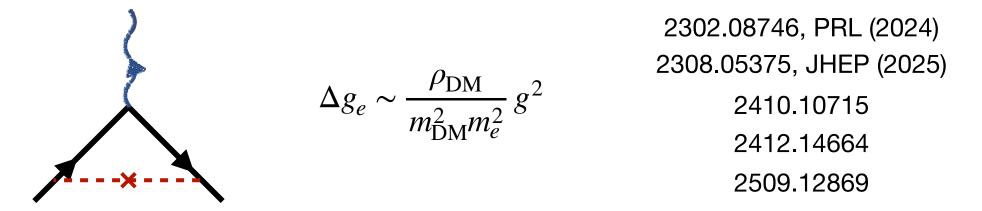
$$J(\mathbf{x}, t) = \partial_t (\mathbf{v} \cdot \hat{\mathbf{s}}) \, \delta^{(3)} (\mathbf{x} - \mathbf{r}(t)).$$

$$d\mathbf{v} - \mathbf{F} - g \, \ddot{a} \, \hat{\mathbf{s}}$$

Classical analogues for ultralight DM effects essentially always exist

"DM Background-Induced" g-2 Shifts are Classical

Claim: DM background yields very strong shifts of electron g-2 and EDMs



Sold as intrinsically quantum, but again can be derived classically!

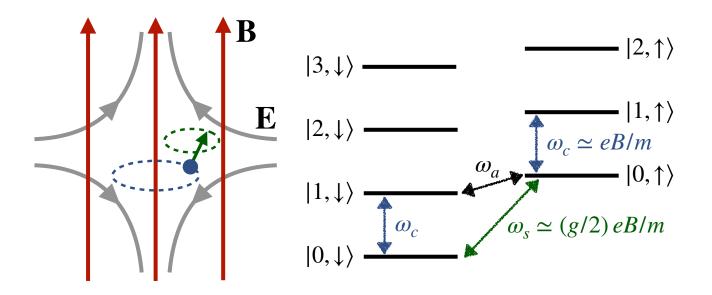
KZ, JHEP (2025)

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad \frac{d\mathbf{S}}{dt} = \mathbf{S} \times \left(\frac{qg_e}{2m_e} \left(\mathbf{B} - \frac{\gamma}{\gamma + 1} (\mathbf{v} \cdot \mathbf{B}) \mathbf{v} - \mathbf{v} \times \mathbf{E}\right) + \frac{\gamma^2}{\gamma + 1} \mathbf{v} \times \mathbf{a}\right).$$

Like the ponderomotive force or gravitational wave memory, derived by carefully solving for classical motion at second order

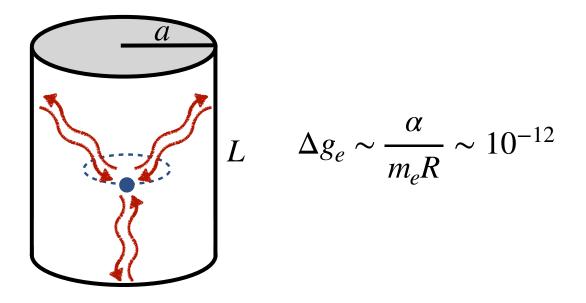
Classical derivation also reveals IR cutoff, making effect negligible in practice

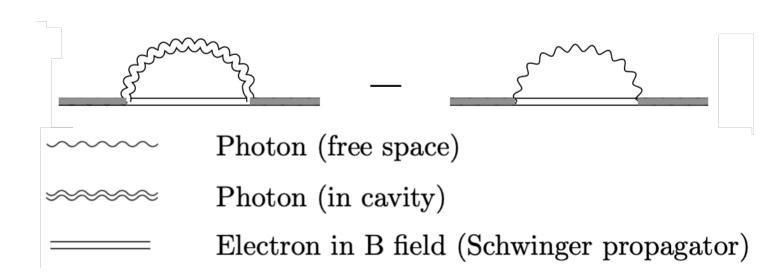
Cavity Induced g-2 Shifts are Classical



Leading electron g-2 experiments compare ω_c and ω_s of a trapped electron

Main theory uncertainty: effect of the cavity around the electron!



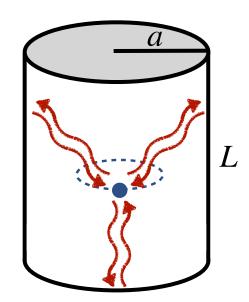


classical: radiation self-field

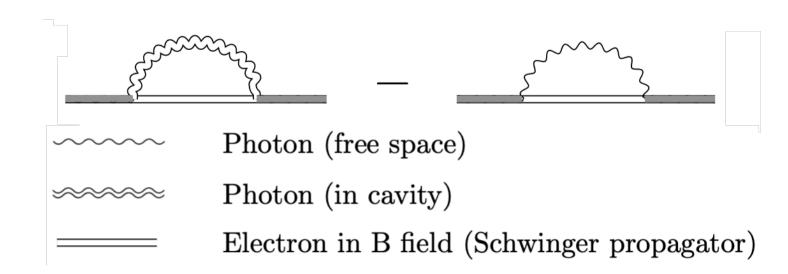
quantum: modification of propagator

Both calculations UV divergent, require renormalization

Cavity Induced g-2 Shifts are Classical

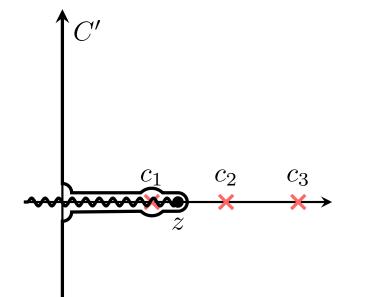


$$\Delta g_e \sim \frac{\alpha}{m_e R} \sim 10^{-12}$$



classical: radiation self-field

quantum: modification of propagator



Quantum effect has never been calculated for a closed cavity

Could include nonclassical finite part from UV matching

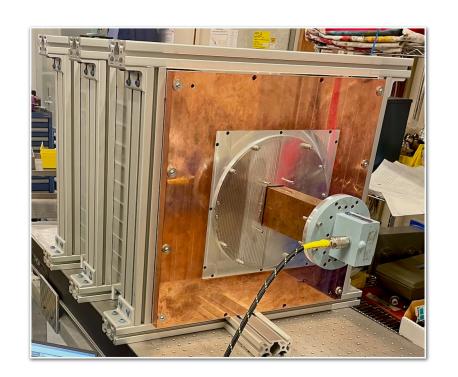
Textbook techniques don't work; developed new method with contour integration; recovers classical result exactly!

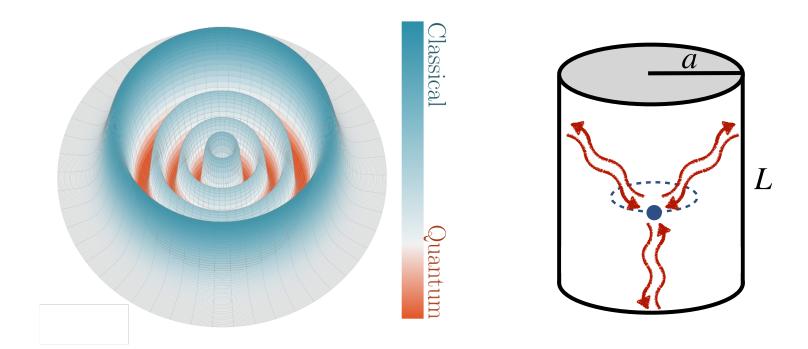
Can improve systematic uncertainty for future measurements

Day, Harnik, Kahn, Pavaskar, KZ, 2511.xxxxx

Conclusion

The job of a theorist is to guide experiment while having fun!





We should find what generic new effects may (or may not) appear in precision measurements, and understand them as clearly as possible

For ultralight DM, quantum effects are not observable even under optimistic assumptions; experiments can proceed classically